The Ba ions, which lie on diad axes, are each surrounded by ten (i.e. five pairs of) O atoms. These are: $\mathrm{O}(4)$ (ketonic) at $2 \cdot 73, \mathrm{O}(6)$ (water) at $2 \cdot 80, \mathrm{O}(2)$ (ketonic) at $2 \cdot 89, \mathrm{O}(1)$ and $\mathrm{O}(3)$ (carboxylic) at $2 \cdot 87$ and $3.06 \AA$ respectively.

Financial assistance from the National Research Council of Canada is gratefully acknowledged.

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# Sodium Neodymium Metaphosphate $\mathrm{NaNdP}_{4} \mathrm{O}_{12}$ 

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(Received 23 January 1976; accepted 12 March 1976)


#### Abstract

Monoclinic, $P 2_{1} / n\left(C_{2 h}^{5}\right) ; a=9.907$ (4), $b=$ $13 \cdot 10$ (1), $c=7 \cdot 201$ (3) $\AA, \beta=90 \cdot 51$ (3) ${ }^{\circ} ; Z=4, D_{m}=$ $3.45, D_{c}=3.43 \mathrm{~g} \mathrm{~cm}^{-3} ; \mu(\mathrm{Mo} K \alpha)=63.8 \mathrm{~cm}^{-1}$. The structural framework consists of helical chains of $\left(\mathrm{PO}_{3}\right)_{\infty}$ along the $c$ axis. $\mathrm{Nd}^{3+}$ and $\mathrm{Na}^{+}$ions alternate linearly in a direction surrounded by four $\left(\mathrm{PO}_{3}\right)_{\infty}$ chains. $\mathrm{NdO}_{8}$ dodecahedra and Na polyhedra form zigzag chains sharing faces. The refinement converged to $R=0.028, R_{w}=0.031$ for 2592 independent observed reflexions.

Introduction. Following the development of an efficient laser material $\mathrm{LiNdP}_{4} \mathrm{O}_{12}$ (Yamada, Otsuka \& Nakano, 1974), another new Nd phosphate crystal $\mathrm{NaNdP}_{4} \mathrm{O}_{12}$ was found in our laboratory (Nakano, Otsuka \& Yamada, 1976). The cell dimensions of the crystal are similar to those of the other alkaline Nd metaphosphate previously reported (Table 1). Specimens were selected from crystals grown from


## Table 1. The axial relations for three alkaline Nd metaphosphates

| Crystal | Space group | $a$ | $b$ | c | $\beta$ | $Z$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathrm{LiNdP}_{4} \mathrm{O}_{12}^{(a)}$ | I2/c | 9.844 Å | 7.008 | A 13.25 Å | $90.1^{\circ}$ | 4 |
| $\mathrm{LiNdP}_{4} \mathrm{O}_{12}^{(b)}$ | C2/c | $16 \cdot 408$ | 7.035 | 9.729 | $126 \cdot 38$ | 4 |
| $\mathrm{NaNdP} \mathrm{C}_{4} \mathrm{O}_{12}^{(c)}$ | $P 2_{1} / n$ | 9.907 | $13 \cdot 10$ | 7.201 | 90.51 | 4 |
| K $\mathrm{NdP}_{4} \mathrm{O}_{12}^{(d)}$ | $P 2_{1}$ | 7.266 | 8.436 | 8.007 | 91.97 | 2 |

(a) Koizumi (1976); (b) Hong (1975a); (c) this work; (d) Hong (1975b).
the melt of $33 \mathrm{Na}_{2} \mathrm{O}-7 \mathrm{Nd}_{2} \mathrm{O}_{3}-60 \mathrm{P}_{2} \mathrm{O}_{5}$ by the Kyropoulos technique (Nakano et al., 1976). Precession and Weissenberg photographs exhibited $2 / m$ Laue symmetry with the following systematic absences: $h 0 l$ when $h+l=2 n+1,0 k 0$ when $k=2 n+1$.

A prismatic crystal $0.15 \times 0.20 \times 0.65 \mathrm{~mm}$ was mounted with its $c$ axis along the $\varphi$ axis of the Rigaku Denki automatic four-circle X-ray diffractometer. Reflexions within $(\sin \theta / \lambda)=0.72 \AA^{-1}$ were measured using the $\omega-2 \theta$ scan technique with a Ge monochromator. The intensities were corrected for Lorentzpolarization, absorption and extinction effects.

The location of atoms, except oxygen, was determined by the three-dimensional Patterson method assuming a similarity to the $\mathrm{LiNdP}_{4} \mathrm{O}_{12}$ structure (Koizumi, 1976). Structure factors based on the Patterson coordinates gave the conventional $R$ value of $0 \cdot 283$. Successive Fourier syntheses clearly revealed the oxygen positions indicating tetrahedral coordination around the P atom.

All the atomic parameters were refined by the fullmatrix least-squares method (Busing, Martin \& Levy, 1962) to give $R=0.028, R_{w}=0.031$. The final atomic parameters are listed in Table 2.*

* A list of structure factors has been deposited with the British Library Lending Division as Supplementary Publication No. SUP 31747 (13 pp., 1 microfiche). Copies may be obtained through The Executive Secretary, International Union of Crystallography, 13 White Friars, Chester CH 11 NZ, England.

Table 2. Positional and thermal parameters obtained in the final least-squares refinement
Estimated standard deviations are given in parentheses. Anisotropic thermal factors ( $\times 10^{5}$ ) are given by the expression:
$T=\exp \left[-\left(h^{2} \beta_{11}+k^{2} \beta_{22}+l^{2} \beta_{33}+2 h k \beta_{12}+2 k l \beta_{23}+2 h l \beta_{13}\right)\right]$.

|  | $x$ | $y$ | $z$ | $\beta_{11}$ | $\beta_{22}$ | $\beta_{33}$ | $\beta_{12}$ | $\beta_{13}$ | $\beta_{23}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Nd | 0.0238 (1) | 0.2174 (1) | $0 \cdot 9868$ (1) | 301 (2) | 180 (1) | 523 (3) | 4 (1) | 15 (1) | -1 (1) |
| $\mathrm{P}(1)$ | $0 \cdot 1943$ (1) | $0 \cdot 3722$ (1) | $0 \cdot 3548$ (2) | 343 (9) | 178 (5) | 553 (17) | 3 (5) | 0 (10) | 0 (7) |
| $\mathrm{P}(2)$ | $0 \cdot 2561$ (1) | $0 \cdot 4001$ (1) | $0 \cdot 7509$ (2) | 366 (9) | 174 (5) | 552 (17) | -13 (6) | 8 (10) | -1 (7) |
| $\mathrm{P}(3)$ | $0 \cdot 3037$ (1) | $0 \cdot 0896$ (1) | $0 \cdot 2305$ (2) | 371 (9) | 175 (5) | 574 (17) | 19 (5) | 28 (10) | -7 (8) |
| $\mathrm{P}(4)$ | $0 \cdot 2360$ (1) | $0 \cdot 1145$ (1) | $0 \cdot 6197$ (2) | 348 (9) | 179 (5) | 592 (17) | -6 (5) | 62 (10) | 8 (7) |
| Na | $0 \cdot 4338$ (2) | 0.2787 (2) | 0.0001 (3) | 553 (21) | 454 (14) | 964 (38) | 37 (14) | -75 (23) | 145 (19) |
| $\mathrm{O}(1)$ | $0 \cdot 1049$ (3) | 0.2897 (3) | 0.2845 (4) | 481 (31) | 215 (17) | 649 (53) | -54 (19) | -60 (33) | -45 (25) |
| $\mathrm{O}(2)$ | $0 \cdot 3417$ (3) | 0.3671 (3) | 0.3192 (4) | 387 (29) | 261 (18) | 707 (54) | 53 (18) | 30 (32) | 73 (25) |
| $\mathrm{O}(3)$ | $0 \cdot 2055$ (4) | 0.3350 (3) | 0.9057 (5) | 483 (5) | 227 (19) | 748 (58) | -72 (19) | 29 (34) | 74 (25) |
| $\mathrm{O}(4)$ | 0.4035 (3) | 0.3937 (3) | 0.7143 (5) | 361 (29) | 230 (17) | 749 (56) | -11 (18) | 17 (32) | - 58 (25) |
| O(5) | $0 \cdot 1327$ (3) | $0 \cdot 4782$ (2) | 0.2834 (5) | 433 (31) | 164 (16) | 933 (62) | 1 (18) | -55 (35) | 68 (25) |
| O (6) | 0.1674 (3) | 0.3769 (3) | $0 \cdot 5718$ (4) | 363 (30) | 256 (18) | 614 (54) | -11 (19) | -115 (32) | -31 (25) |
| $\mathrm{O}(7)$ | $0 \cdot 2120$ (4) | $0 \cdot 1102$ (3) | 0.0701 (5) | 545 (33) | 273 (19) | 641 (55) | 106 (20) | -74 (34) | -2 (26) |
| $\mathrm{O}(8)$ | 0.4180 (3) | $0 \cdot 1607$ (3) | 0.2614 (5) | 522 (33) | 240 (18) | 732 (58) | -112 (20) | 60 (34) | - 55 (26) |
| O(9) | $0 \cdot 3470$ (3) | $0 \cdot 1896$ (3) | $0 \cdot 6328$ (5) | 449 (31) | 211 (17) | 721 (56) | -88 (18) | 81 (33) | -37 (25) |
| $\mathrm{O}(10)$ | $0 \cdot 1016$ (3) | $0 \cdot 1418$ (3) | $0 \cdot 6929$ (5) | 414 (30) | 253 (18) | 712 (55) | 41 (19) | 82 (32) | -12 (35) |
| $\mathrm{O}(11)$ | $0 \cdot 2908$ (3) | $0 \cdot 0140$ (3) | 0.7134 (5) | 471 (31) | 182 (17) | 850 (58) | 0 (18) | 20 (33) | 38 (25) |
| $\mathrm{O}(12)$ | $0 \cdot 2119$ (3) | $0 \cdot 0806$ (3) | $0 \cdot 4107$ (5) | 455 (31) | 271 (18) | 611 (55) | -70 (19) | 59 (33) | -67 (25) |

Discussion. Bond distances and angles are given in Table 3 and Fig. 3. Views of the structure projected along the $c$ and $b$ axes are shown in Figs. 1 and 2, respectively.

The $\mathrm{NdO}_{8}$ dodecahedron is considerably distorted, as in $\mathrm{NdP}_{5} \mathrm{O}_{14}$ (Hong, 1974). The dodecahedron shares all its oxygen atoms with the corners of neighbouring $\mathrm{PO}_{4}$ tetrahedra and with the faces of neighbouring Na polyhedra (Fig. 2).

The relative arrangement of $\mathrm{Nd}^{3+}$ and $\mathrm{Na}^{+}$resembles that of the other alkaline Nd metaphosphates, in which the displacement of the alkaline ion from the axis of the $\mathrm{Nd}^{3+}-\mathrm{M}^{+}\left(\mathrm{M}^{+}\right.$stands for alkaline metal ion) zigzag chain increases with the ionic radius and coordination number of $\mathrm{M}^{+}$[Table 4, Fig. 3(c)].

The Na polyhedron is also quite irregular, as are the K polyhedra in the $\mathrm{KNdP}_{4} \mathrm{O}_{12}$ structure (Hong, 1975b). Five O atoms constitute the first coordination sphere around the $\mathrm{Na}^{+}$ion at distances approx-


Fig. 1. Projection of the $\mathrm{NaNdP}_{4} \mathrm{O}_{12}$ structure along [010].
imately equal to the sum of the atomic radii, $2 \cdot 40 \sim$ $2.57 \AA$. The sixth bond makes the polyhedron into an octahedron and is substantially longer: $2.72 \AA$

Table 3. Bond distances $(\AA)$ and angles $\left({ }^{\circ}\right)$ with standard errors in $\mathrm{NaNdP}_{4} \mathrm{O}_{12}$

| Dodecahedron around |  |
| :--- | ---: |
| Nd [for Nd-O see Fig. 3(a)] |  |
| O(1)-O(3) | $2.965(5)$ |
| $\mathrm{O}(1)-\mathrm{O}(7)$ | $3.002(5)$ |
| $\mathrm{O}(1)-\mathrm{O}(9)$ | $2.801(4)$ |
| $\mathrm{O}(2)-\mathrm{O}(4)$ | $2.936(5)$ |
| $\mathrm{O}(2)-\mathrm{O}(8)$ | $2.838(6)$ |
| $\mathrm{O}(2)-\mathrm{O}(10)$ | $2.728(4)$ |
| $\mathrm{O}(3)-\mathrm{O}(8)$ | $3.041(5)$ |
| $\mathrm{O}(4)-\mathrm{O}(9)$ | $2.795(6)$ |
| $\mathrm{O}(7)-\mathrm{O}(10)$ | $2.972(5)$ |
| $\mathrm{O}(8)-\mathrm{O}(9)$ | $2.785(5)$ |

Tetrahedron around $\mathrm{P}(1)$

| $\mathrm{P}(1)-\mathrm{O}(1)$ | $1.489(4)$ |
| :--- | :--- |
| $\mathrm{P}(1)-\mathrm{O}(2)$ | $1.482(3)$ |
| $\mathrm{P}(1)-\mathrm{O}(5)$ | $1.603(3)$ |
| $\mathrm{P}(1)-\mathrm{O}(6)$ | $1.584(3)$ |
| $\mathrm{O}(1)-\mathrm{P}(1)-\mathrm{O}(2)$ | $119 \cdot 8(2)$ |
| $\mathrm{O}(1)-\mathrm{P}(1)-\mathrm{O}(5)$ | $106 \cdot 2(2)$ |
| $\mathrm{O}(1)-\mathrm{P}(1)-\mathrm{O}(6)$ | $105 \cdot 6(2)$ |
| $\mathrm{O}(2)-\mathrm{P}(1)-\mathrm{O}(5)$ | $111 \cdot 2(2)$ |
| $\mathrm{O}(2)-\mathrm{P}(1)-\mathrm{O}(6)$ | $109.2(2)$ |
| $\mathrm{O}(5)-\mathrm{P}(1)-\mathrm{O}(6)$ | $102 \cdot 8(2)$ |

$\begin{array}{ll}\text { Tetrahedron around } \mathrm{P}(3) \\ \mathrm{P}(3)-\mathrm{O}(5) & 1.593(3) \\ \mathrm{P}(3)-\mathrm{O}(7) & 1.500(4) \\ \mathrm{P}(3)-\mathrm{O}(8) & 1.485(4) \\ \mathrm{P}(3)-\mathrm{O}(12) & 1.582(4) \\ & \\ \mathrm{O}(5)-\mathrm{P}(3)-\mathrm{O}(7) & 110 \cdot 0(2) \\ \mathrm{O}(5)-\mathrm{P}(3)-\mathrm{O}(8) & 106 \cdot 4(2) \\ \mathrm{O}(5)-\mathrm{P}(3)-\mathrm{O}(12) & 102 \cdot 0(2) \\ \mathrm{O}(7)-\mathrm{P}(3)-\mathrm{O}(8) & 118.1(2) \\ \mathrm{O}(7)-\mathrm{P}(3)-\mathrm{O}(12) & 107 \cdot 3(2) \\ \mathrm{O}(8)-\mathrm{P}(3)-\mathrm{O}(12) & 110.9(2)\end{array}$
Polyhedron around Na [for
Na-O see Fig. 3(b)]

| $\mathrm{O}(1)-\mathrm{O}(9)$ | $2.801(4)$ |
| :--- | :--- |
| $\mathrm{O}(2)-\mathrm{O}(8)$ | $2.836(6)$ |
| $\mathrm{O}(2)-\mathrm{O}(10)$ | $2.728(4)$ |
| $\mathrm{O}(3)-\mathrm{O}(4)$ | $2.508(5)$ |
| $\mathrm{O}(3)-\mathrm{O}(9)$ | $3.067(5)$ |
| $\mathrm{O}(4)-\mathrm{O}(9)$ | $2.795(6)$ |
| $\mathrm{O}(8)-\mathrm{O}(9)$ | $2.785(5)$ |
| $\mathrm{Nd}-\mathrm{Nd}$ |  |
|  | $5.719(1)$ |
|  | $6.209(1)$ |

Tetrahedron around $\mathrm{P}(2)$

| $\mathrm{P}(2)-\mathrm{O}(3)$ | $1.487(4)$ |
| :--- | ---: |
| $\mathrm{P}(2)-\mathrm{O}(4)$ | $1.484(3)$ |
| $\mathrm{P}(2)-\mathrm{O}(6)$ | $1.596(3)$ |
| $\mathrm{P}(2)-\mathrm{O}(11)$ | $1.583(4)$ |
|  |  |
| $\mathrm{O}(3)-\mathrm{P}(2)-\mathrm{O}(4)$ | $115.2(2)$ |
| $\mathrm{O}(3)-\mathrm{P}(2)-\mathrm{O}(6)$ | $108.2(2)$ |
| $\mathrm{O}(3)-\mathrm{P}(2)-\mathrm{O}(11)$ | $108.8(2)$ |
| $\mathrm{O}(4)-\mathrm{P}(2)-\mathrm{O}(6)$ | $113.3(2)$ |
| $\mathrm{O}(4)-\mathrm{P}(2)-\mathrm{O}(11)$ | $111.7(2)$ |
| $\mathrm{O}(6)-\mathrm{P}(2)-\mathrm{O}(11)$ | $98.5(2)$ |

Tetrahedron around $\mathrm{P}(4)$

| $\mathrm{P}(4)-\mathrm{O}(9)$ | $1 \cdot 479(3)$ |
| :--- | :--- |
| $\mathrm{P}(4)-\mathrm{O}(10)$ | $1.482(4)$ |
| $\mathrm{P}(4)-\mathrm{O}(11)$ | $1.578(4)$ |
| $\mathrm{P}(4)-\mathrm{O}(12)$ | $1.589(4)$ |
|  |  |
| $\mathrm{O}(9)-\mathrm{P}(4)-\mathrm{O}(10)$ | $117.7(2)$ |
| $\mathrm{O}(9)-\mathrm{P}(4)-\mathrm{O}(11)$ | $105.6(2)$ |
| $\mathrm{O}(9)-\mathrm{P}(4)-\mathrm{O}(12)$ | $111.4(2)$ |
| $\mathrm{O}(10)-\mathrm{P}(4)-\mathrm{O}(11)$ | $112.5(2)$ |
| $\mathrm{O}(10)-\mathrm{P}(4)-\mathrm{O}(12)$ | $105.6(2)$ |
| $\mathrm{O}(11)-\mathrm{P}(4)-\mathrm{O}(12)$ | $103.1(2)$ |

Table 4. The relations between the ionic radius ( $r$ ) and the displacement of the alkaline ion $\left(\mathrm{M}^{+}\right)$
from the axis of the $\mathrm{Nd}^{3+}-\mathrm{M}^{+}$zigzag chain (d) in the $\mathrm{MNdP}_{4} \mathrm{O}_{12}$ structure

|  | $\mathrm{Li}^{+}$ | $\mathrm{Na}^{+}$ | $\mathrm{K}^{+}$ |
| :--- | :--- | :--- | :--- |
|  | 0.60 | 0.95 | 1.33 |
| $r(\AA)$ | 0 | 0.46 | 0.79 |
| $d(\AA)$ |  |  |  |
| Coordination  <br> $\quad$ number $(n)$  <br> numer  | 4 | $6+1$ | 7 |

(International Tables for $X$-ray Crystallography, 1962). Another oxygen atom $[\mathrm{O}(9)$ ] lies at a distance of $3.02 \AA$ [Fig. 3(b)]. This type of coordination around $\mathrm{Na}^{+}$can also be seen in the structures of $\left[\mathrm{Na}_{3} \mathrm{H}\left(\mathrm{PO}_{3}\right)_{4}\right]_{x}$ (Jost, 1968) and $\mathrm{Na}_{2} \mathrm{ZrSiO}_{5}$ (Treushnikov, Ilyukhin \& Belov, 1970). The weaker bonds to $\mathrm{Na}^{+}$cause a larger thermal motion ( $B_{\mathrm{eq}}^{\mathrm{Na}}: 2 \cdot 43$ ) compared with those of the other atoms ( $B_{\mathrm{eq}}: 1 \cdot 17 \sim 1.78$ ). Such tendencies are also reported in the above-mentioned structures.

The helical $\left(\mathrm{PO}_{3}\right)_{\infty}$ chains in the $\mathrm{NaNdP}_{4} \mathrm{O}_{12}$ structure are similar to those of the other alkaline Nd metaphosphates, in which the chains run along the shortest unit-cell directions. In the present structure the chain axis deviates considerably from the centre of each quarter cell and results in the formation of a larger tunnel along the $c$ axis than is found in the $\mathrm{LiNdP} \mathrm{C}_{4} \mathrm{O}_{12}$ structure.

It is to be noted that the adjacent $\mathrm{NdO}_{8}$ polyhedra do not share any oxygen atom as in the structures of $\mathrm{LiNdP}_{4} \mathrm{O}_{12}$ and $\mathrm{KNdP}_{4} \mathrm{O}_{12}$. Although the shortest $\mathrm{Nd}-\mathrm{Nd}$ distance ( $5.719 \AA$ ) is somewhat longer than that in $\mathrm{LiNdP}_{4} \mathrm{O}_{12}(5 \cdot 644 \AA)$, the laser characteristics of the present crystal are less prominent than the latter (Hong, 1975a). This is related to the lower symmetry of the $\mathrm{NdO}_{8}$ dodecahedron and $\left(\mathrm{PO}_{3}\right)_{\infty}$ chains and, moreover, the larger interaction between $\mathrm{Nd}^{3+}$ and the alkaline ion $\left(\mathrm{Na}^{+}\right)$.

The author is indebted to Mr J. Nakano who supplied the crystals and Mr T. Ogawa of Rigaku Denki Co., for the intensity measurements. He also wishes to express his gratitude to Dr N. Niizeki and Dr H. Iwasaki for their helpful discussions and encouragement.

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Fig. 2. Schematic view of the $-\mathrm{Nd}^{3+}-\mathrm{Na}^{+}-$chain in the [010] direction.


Fig. 3. Schematic illustration of (a) the $\mathrm{NdO}_{8}$ dodecahedron, (b) the sodium coordination polyhedron and (c) the $-\mathrm{Nd}^{3+}-\mathrm{M}^{+}$zigzag chain in alkaline Nd metaphosphate.

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